The Mathematical Association of Victoria

MATHEMATICAL METHODS and MATHEMATICAL METHODS (CAS)

Trial written examination 1

2006

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Mathematical Methods and Mathematical Methods (CAS) Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Working space

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider $f: R\backslash\{3\} \rightarrow R$, where $f(x) = \frac{2}{x-3} + 4$.

a. Find the rule and domain for f^{-1}.

b. Find the coordinates of the points where $f = f^{-1}$.

3 + 2 = 5 marks

Question 2

Solve the equation $2\sin^2(x) = 1$ for $x \in [-\pi, \pi]$, expressing all solutions as exact values.

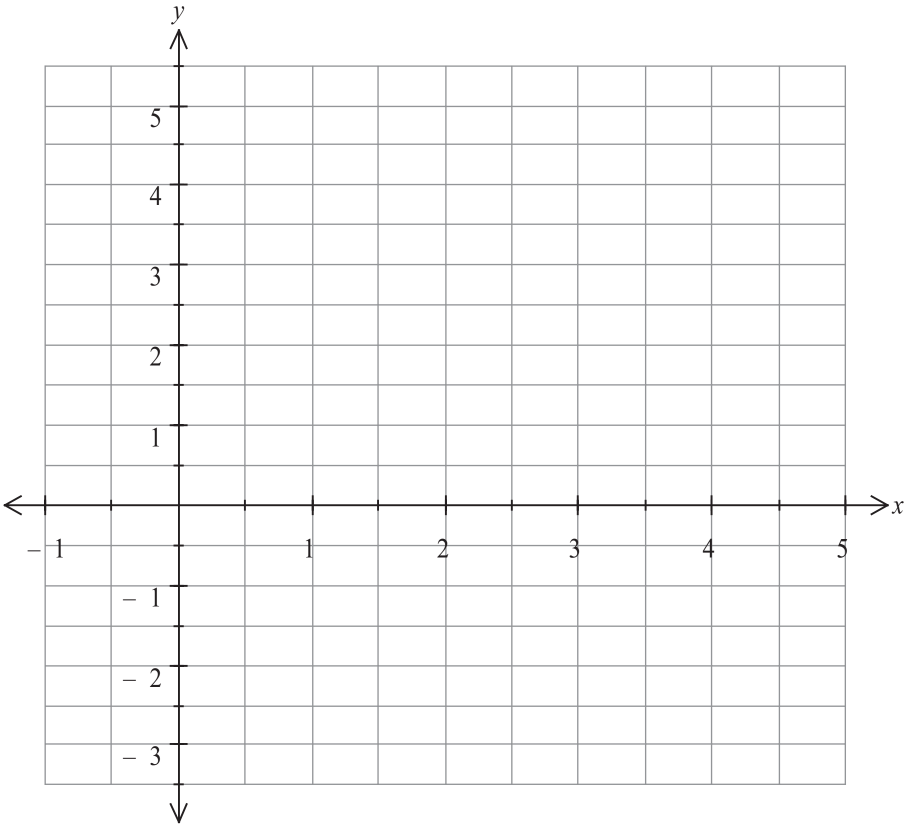
3 marks

TURN OVER

Question 3

Consider $f:(0,4] \to R$, where $f(x) = (x-2)^{\frac{1}{3}}$ and $g:[0,4) \to R$, where $g(x) = (x-2)^{\frac{1}{5}}$.

a. Sketch the graphs of f and g on the set of axes below.



b. Hence, sketch the graph of $f+g$ on the set of axes in part **a**.

c. Use calculus to find the area bounded by the graphs of f and g.

3 + 1 + 3 = 7 marks

Question 4

Consider $f: R\backslash\{0\} \to R$, where $f(x) = \log_e(|x|)$ and $g:\left(-\infty, \frac{-5}{2}\right) \to R$, where $g(x) = 2x + 5$.

a. Find the rule and the domain for $f(g(x))$.

b. Find the equation of the normal to $f(g(x))$ at $x = -4$.

2 + 3 = 5 marks

Question 5

Water is being poured into a cylindrical vase at a rate of 8 cm^3/s. The height of the vase is double the radius.

a. Find *V*, the volume of the vase, in terms of *h*, the height of the vase.

b. At what rate is the water level rising when the depth is 2 cm?

1 + 3 = 4 marks

Question 6

Consider $f:(-1,\infty) \to R$, where $f(x) = \dfrac{2e^{3x}}{\sqrt{x+1}}$.

a. Find $f'(x)$ in the form $\dfrac{(ax+b)e^{cx}}{(x+1)^{d}}$, where a, b, c and d are real constants.

b. Hence state the nature and the coordinates of the stationary point of f.

3 + 3 = 6 marks

Question 7

In a particular town, both unleaded petrol and ethanol-blended petrol are sold at the petrol station. A petrol price rise will take place on 1st June. It is predicted that during the month of June, 20% of drivers who bought unleaded petrol the previous week will buy the cheaper ethanol-blended petrol the following week. It is also predicted that only 10% of drivers who buy ethanol-blended petrol each week in June will buy unleaded petrol the following week.

A driver is chosen at random from the group that bought unleaded petrol in the last week of May. According to the predictions, what is the probability that the chosen driver will buy ethanol-blended fuel in the **second** week of June? (Assume that all of the drivers in the group buy petrol every week).

3 marks

Question 8

A continuous random variable, X, has the probability density function

$$f(x) = \begin{cases} k\cos\left(\frac{x}{2}\right) & \text{for } 0 \le x \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

where k is a positive real constant.

a. Evaluate $\int_0^{\pi} \cos\left(\frac{x}{2}\right)dx$.

b. Hence find the value of k.

c. Differentiate $x\sin\left(\frac{x}{2}\right) + 2\cos\left(\frac{x}{2}\right)$.

d. Hence evaluate $\int_0^{\pi} x\cos\left(\frac{x}{2}\right)dx$.

e. Hence find $E(X)$, the expected value of X.

2 + 1 + 2 + 1 + 1 = 7 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Version 2 – March 2006

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

Version 2 – March 2006